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ADMM Based Regularized Optimization for Reconstruction of Digital In-line Holography

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Abstract: Compressed sensing is effective in twin image removal of in-line holograms, but has slower convergence and higher computational time. Proposed method suppresses twin images by solving an inverse problem using ADMM and offers better convergence.

1. Introduction

In digital in-line holography (DIH), a hologram is formed from the interference of a reference field in alignment with an object field. The intensity of this hologram is given by,

$$I_H = |Ob+Re|^2 = |Ob|^2 + |Re|^2 + Ob^*Re + ObRe^* \tag{1}$$

$$= |Ob|^2 + |Re|^2 + V(x,y) + V^*(x,y) \tag{2}$$

where, V(x, y) = Ob*Re forms the twin image. Re(x, y) is the reference wave, Ob(x, y) is the object wave, and (x, y) is the spatial coordinates. Reference field $|Re|^2$ is assumed to be without loss and can be removed from the hologram. $|Ob|^2$ is regarded as the noise term. V(x, y) = Ob*Re represents an out-of-focus conjugate, which smears the reconstructed image. This blurring or poor resolution in the reconstruction of 3D shapes hinders accurate information retrieval. Numerical methods are widely used in twin image removal. Twin image can be eliminated by solving regularized optimization problem through compressed sensing approach [1] where Two-step Iterative Shrinkage Thresholding (TwIST) algorithm was used. The method was found to be more effective than previous phase retrieval method [2]. But it suffers from slower convergence and thus higher computational time. The proposed method offers better results and faster convergence using the Alternating Direction Method of Multipliers (ADMM) [3] algorithm.

2. Methodology

The 3D reconstruction can be formulated as an inverse optimization problem.

$$\hat{\beta} = \underset{\beta}{argmin} \frac{1}{2} \|b - A\beta\|_2^2 + \lambda \|\beta\|_{TV} \tag{3}$$

where $\hat{\beta}$ represents approximation of object field that minimizes the objective function in Eq (3). b is the recorded hologram. $A\hat{\beta}$ is obtained by forward propagation and is the real part of the complex field obtained from the mapping of object field to the scattered object wavefront. λ represents regularization parameter. Total Variation (TV) norm is the regularization term. TV norm is computed as

$$\|\beta\|_{TV} = \|\nabla\beta\|_1 = \sum_{i=1}^{N_x}\sum_{j=1}^{N_y} |(\beta(i,j) - \beta(i-1,j)) + (\beta(i,j) - \beta(i,j-1))| \tag{4}$$

Reformulate Eq (3) in ADMM as

$$\underset{\beta}{argmin} \frac{1}{2} \|z\|_2^2 + \lambda \|y\|_1 \tag{5}$$

subject to: $z = A\beta - b$ and $y = \nabla\beta$
Writing Augmented Lagrangian in terms of scaled dual variables,

$$\mathfrak{L}(\beta, z, m, n) = \frac{1}{2}\|z\|_2^2 + \lambda \|y\|_1 + \frac{\eta}{2}\left\|z + b - A\beta + \frac{m}{\eta}\right\|_2^2 + \frac{\zeta}{2}\left\|y - \nabla\beta + \frac{n}{\zeta}\right\|_2^2 \tag{6}$$

m,n are Lagrange multipliers and η, ζ are penalty parameters. Eq (6) can be reduced to the following sub-problems

$$\beta^{k+1} = \underset{\beta}{argmin} \frac{\eta}{2} \left\| z + b - A\beta + \frac{m}{\eta} \right\|_2^2 + \frac{\zeta}{2} \left\| y - \nabla\beta + \frac{n}{\zeta} \right\|_2^2 \tag{7}$$

$$z^{k+1} = \underset{z}{argmin} \frac{1}{2} \|z\|_2^2 + \frac{\eta}{2} \left\| z + b - A\beta + \frac{m}{\eta} \right\|_2^2 \tag{8}$$

$$y^{k+1} = \underset{y}{argmin} \lambda \|y\|_1 + \frac{\zeta}{2} \left\| y - \nabla\beta + \frac{n}{\zeta} \right\|_2^2 = S_{\frac{\lambda}{\zeta}} (\nabla\beta^{k+1} - \frac{n^k}{\zeta}) \tag{9}$$

$$m^{k+1} = m^k + \eta(z^{k+1} - A\beta^{k+1} + b) \tag{10}$$

$$n^{k+1} = n^k + \zeta(y^{k+1} - \nabla\beta^{k+1}) \tag{11}$$

The algorithm stops when a predefined tolerance or number of iterations is reached.

3. Results

The effectiveness of the proposed method is verified numerically and experimentally. The results are validated in terms of Mean Square Error (MSE), Structural Similarity Index (SSIM), Peak Signal to Noise Ratio (PSNR) and computation time.

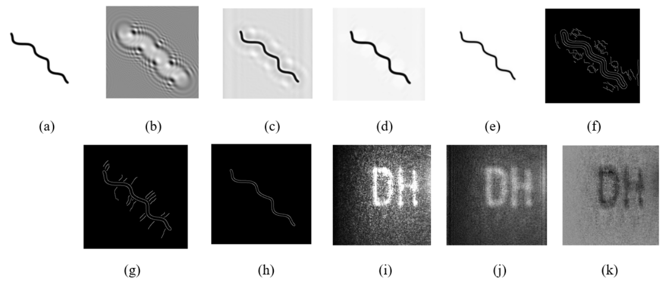

Fig. 1. Reconstruction of holograms.(a) original object.(b) normalized hologram.(c) Amplitude reconstruction after 25 iterations of [1].(d) Amplitude reconstruction after 200 iterations of [1].(e) Amplitude reconstruction after 25 iterations of proposed method. (f) Edge map after 25 iterations of [1] (g) Edge map after 200 iterations of [1].(h) Edge map after 25 iterations of proposed method.(i) Experimental hologram recorded at wavelength of 532 nm. (j) Amplitude reconstruction using [1]. (k) Amplitude reconstruction using proposed method.

Table 1. Performance comparison with [1] for λ=0.1

	Existing [1]	Proposed	Existing [1]	Proposed	Existing [1]	Proposed
Iterations	25	25	50	50	200	200
Time	16.52 sec	4.97 sec	8.75 sec	23.52 sec	81.73sec	31.68 sec
MSE	0.0393	0.000041	0.0383	0.000035	0.0035	0.000019
SSIM	0.93075	0.9954	0.96359	0.99568	0.99212	0.99669
PSNR	22.43dB	43.87dB	23.213dB	44.55dB	23.08dB	47.31dB

The proposed method gives better qualitative results for numerical and experimental holograms. As shown in Table 1, the proposed system performs better in terms of MSE, SSIM and PSNR at a better computation time.

References

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